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STRUCTURE AND DYNAMICS OF THE SLOW PRESSURE WAVE IN A POROUS MEDIUM SATURATED WITH A LIQUID CONTAINING GAS BUBBLES

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A linear-equation system has been derived [1] for a three-phase mixture (porous medium, liquid, and gas bubbles), which incorporates dispersion effects due to bubble oscillations in the waves. An equation system has been given [2] for finite-amplitude waves that incorporates the nonlinearity introduced by the gas-bubble oscillations and by the viscous damping due to the radial motion of the liquid around the oscillating bubbles. Evolutionary equations have been derived [3] for fast and slow waves in a porous medium saturated with a liquid containing gas bubbles, and the evolution and structure of the fast pressure wave were examined by experiment.

We have made an experimental study of the slow-wave evolution and structure in a porous medium saturated with a liquid containing gas bubbles. If it is assumed that the nonlinear, dispersion, and dissipative terms in the [3] system are small, one gets a slow-wave evolutionary equation:

$$\frac{\partial^2 p_c}{\partial t^2} - c_s^2 \frac{\partial^2 p_c}{\partial x^2} + \frac{m\nu}{\alpha K_0}\frac{\partial p}{\partial t} - \frac{R_s}{c_0^2}\left(B\frac{\partial^2 (\delta p_c)^2}{\partial t^2} + \frac{4\nu^*}{3\varphi_0}\frac{\partial^3 p_c}{\partial t^3} + \beta\frac{\partial^4 p_c}{\partial t^4}\right) = 0,$$

$$R_s = \frac{c_s^2}{c_f^2 - c_s^2}\left(\frac{\alpha (MH - c^2)}{c_s^2 (\alpha\rho_0\rho_{c0}/m - \rho_{c0}^2)} - \frac{(\alpha\rho_0 - m\rho_{c0}) M}{(\alpha\rho_0\rho_{c0}/m - \rho_{c0}^2)}\right). \quad (1)$$

Here p_c is the pressure in the gas–liquid mixture, c_f and c_s are the speeds of the fast and slow waves, c_0 the speed of sound in the mixture, B the nonlinearity coefficient, β the dispersion coefficient, ν^* the effective viscosity, m porosity, K_0 permeability, φ_0 volume gas content, and α the adjoint-mass coefficient. All the symbols correspond to those given in [3]. The expressions for c_f, c_s, and so on are defined in [3]. When one neglects the dissipative losses in (1), one gets a stationary solution: solitons [4]. The soliton velocity and half-width are

$$\frac{V_c}{c_s} = \left(1 + \frac{\gamma+1}{3\gamma}\frac{\delta p_c}{p_0} R_s\right)^{1/2}, \quad \delta = \left(\beta\left(4 + \frac{12 p_0}{\delta p_c}\frac{\gamma}{\gamma+1}\right)\right)^{1/2}\frac{c_s}{c_0}.$$

If we neglect the viscous dissipation in (1) due to the longitudinal displacement of the liquid and porous skeleton in the wave, i.e., if we neglect the term containing $\partial/\partial t$, we get a stationary solution: a shock wave [4]. The shock-wave speed is

$$\frac{V}{c_s} = \left(1 + \frac{\gamma+1}{2\gamma}\frac{\delta p_c}{p_0} R_s\right)^{1/2}. \quad (2)$$

We used a shock tube apparatus [2]. The medium was a random packing of sintered lucite spheres about 2 mm in diameter, which were sintered directly in the working part. The liquid was distilled water and the gases were air and carbon

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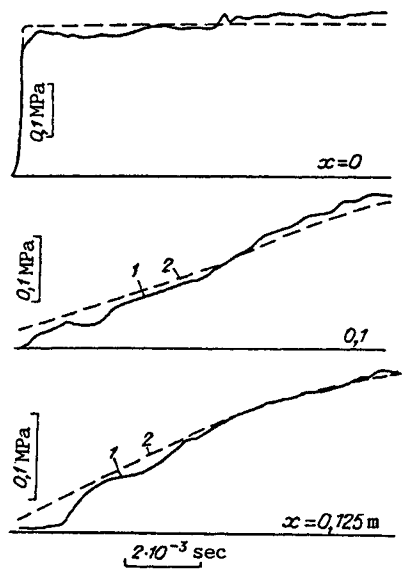

Fig. 1

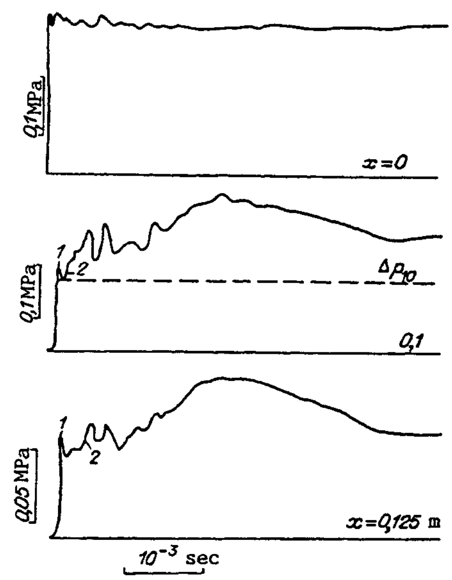

Fig. 2

dioxide. Piezoelectric detectors were placed along the working section; they did not touch the skeleton of the porous medium and measured the pressure-wave profile in the liquid.

There are qualitative changes in slow-wave structure as $(K_b + (4/3)\mu)/K_c$ varies (K_b and μ are the bulk modulus and shear modulus of the porous skeleton, while K_c is the elastic modulus of the gas–liquid mixture). Then $(K_b + (4/3)\mu)/K_c \gg 1$ corresponds to a fairly high gas content ($\varphi \gg 1\%$), when the dispersion and nonlinear effects have very little effect on the slow-wave propagation. The main mechanism governing the wave structure is viscous dissipation due to longitudinal displacement of the liquid and porous skeleton in the wave. Figure 1 shows the evolution of a stepped pressure wave in the porous medium saturated with water containing air bubbles (line 1). The parameters of the porous medium ($K_b = 0.8 \cdot 10^9$ N/m^2, m = 0.35, $K_0 = 2.3 \cdot 10^{-9}$ m^2) were the same in all the experiments. The parameters of the saturating liquid were as follows: static pressure $p_0 = 2.3 \cdot 10^5$ Pa, $\varphi_0 = 5.9\%$, and x the distance from the inlet to the porous medium at the point of measurement. The dissipation produces considerable flattening in the slow-wave leading edge. Line 2 shows the result from the diffusion equation to which (1) reduces for low-frequency slow waves [2]. The diffusion approximation describes the slow-wave structure closely. There is a considerable deviation in the calculated curve at the start because the approximate solution gives an infinite propagation speed. The speed calculated from (1), $c_s = 364$ m/sec, agrees with the experimental $V_s = 382$ m/sec within the accuracy of the wave-speed measurement.

If we reduce the gas content to about 1% [when $K_b + (4/3)\mu)/K_2 \sim 1$], the slow-wave structure becomes essentially different. The propagation is substantially affected by the nonlinearity and dispersion, which lead to oscillations at the leading edge. Figure 2 shows the evolution for $p_0 = 5 \cdot 10^5$ Pa and $\varphi_0 = 0.3\%$. Fast and slow waves are generated (lines 1 and 2) from an initial step wave. The fast-wave amplitude corresponds to the calculated Δp_{10} from the Biot model (dashed line), while the measured fast-wave speed $V_f = 1690$ m/sec agrees with that calculated as in [3] within the measurement accuracy. The leading edge of the slow wave has a prominent oscillation. The dissipation damps out the oscillations as the wave propagates. The measured speed for the leading edge is $V_s = 512$ m/sec, which agrees with that calculated from (2).

We examined how the dissipation mechanism arising from heat transfer between the gas and the liquid affected the slow-wave damping in propagation experiments with carbon dioxide as the gas. The thermal conductivity of CO_2 is less than that of air by almost a factor 2, so the heat transfer for CO_2 bubbles is much less. The wave profiles did not differ qualitatively from those shown in Fig. 2, but the slow-wave damping was less than with air. This means that slow-wave evolution calculations must incorporate the heat transfer between the gas and liquid in (1).

Figure 3 shows V_s, the speed of the slow-wave leading edge, as a function of the initial intensity Δp_s for R_s = 0.95-1; 0.85-0.95; 0.75-0.85 (points 1-3). It also shows the shock-wave speed V_s calculated from (2) for R_s = 0.975; 0.9; 0.8 (lines 4-6), which describe the experimental data satisfactorily. The largest deviation in the experimental values occurs for the least

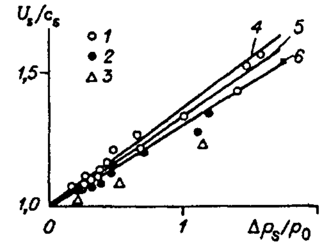

Fig. 3

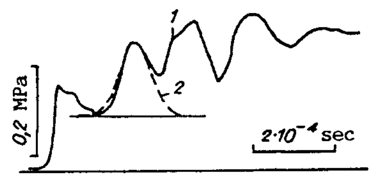

Fig. 4

R_s because at those R_s, the radii of the bubbles in the experiments were minimal and the gas thermal relaxation time became comparable with the characteristic wave period. Consequently, the behavior of the gas and the bubbles deviated considerably from adiabatic.

Figure 4 shows the oscillating structure in the slow-wave leading edge with air as the gas for $p_0 = 4 \cdot 10^5$ Pa and $\varphi_0 = 0.4\%$ (line 1). The first slow-wave oscillation is superimposed on the shape of the soliton having the same intensity as calculated from (1) (line 2). The first slow-wave oscillation is closely described by the soliton shape.

Bubble oscillations are thus responsible for the oscillating structure in the slow-wave leading edge. Also, the slow-wave evolution can be derived from (1) with allowance for the heat transfer between the gas and the liquid in the case of high-permeability porous media, where dissipative effects due to longitudinal displacement of the liquid and the solid skeleton are small.

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